SPORT ARCHER AND BOW INTERACTION

Zanevsky I.P.

Lviv State Institute of Physical Culture,

Casimir Pulaski Technical University (Radom, Poland)

Анотація. Запропоновано метод механіко-математичного й комп'ютерного моделювання взаємодії стрільця з луком під час спільного руху стріли з тятивою. Модель тіла стрільця представлено складним осцилятором з інерційними та в'язко-пружними елементами. Результати подано у формі, прийнятній для використання у навчально-тренувальному процесі спортсменів лучників.

Ключові слова: *спортивна стрільба з лука, біомеханіка, математичне й комп'ютерне моделювання.*

Statement of the problem. During the rapid an arrow is in common space motion with a bow and an archer. The main motion in the vertical plane is caused by the bow grip-limbs-string system. Simultaneously the arrow through the string nock point is involved by the whole system in deflection out of the vertical plane. This rapid motion causes arrow deflection in the lateral plane.

A rigorous three-dimensional analysis of the space system is very complicated and with the other assumptions is not essential. The main part of potential energy, stored in bow limbs, transfers to kinetic energy of the longitudinal motion of the arrow and the string common motion. Some negligible part of it transfers to the deflect motion of the arrow. Although the arrow is in a space motion of the whole system, the problem can be idealised and reduced with two separate systems. The first one is in the vertical plane and the second is in the lateral plane.

The research was done according the plan of collaboration between Lviv State Institute of Physical Culture and Casimir Pulaski Technical University.

Overview of the resent researches and publications. The first step towards to understanding arrow behaviour in internal ballistics was related with explanation of phenomenon that near two centuries is known as the archer's paradox. The investigation of the archer's paradox was found by means of a high speed spark photography which has been undertook in order to secure direct evidence of what an arrow does as it leaves the bow Klopsteg [3]. The archer's paradox is the fact that an arrow does not fly to its mark along the line represented by its axis. The forces played on the arrow during its release do not quite coincide with this axis. The string force plays on the arrow in the bow plane. In a starting position the arrow does not lie in this plane, its axis makes an angle of a few degrees with it. Even the case the nock and head points of the arrow do lie in the bow plane, the longitudinal arrow axis of the string force line does not coincide quit with the bow plane because the initial shape of the arrow axis is not quit straight. So, the string force line does not coincide with the line of cross-section centres of the arrow. The released string pushes the arrow's nock point in the bow plane. Therefore the arrow will move forwards and slightly turn decreasing the angle with this plane. The impulse normal to the axis of the arrow caused by the release of the fingers from the string, as well as the column-like force of the string push the arrow during its acceleration motion results in significant bending of the arrow shaft as it transits the bow. All these factors allow the arrow to undulate around the bow handle and to follow a straight course towards its target without striking the bow handle. Mechanical and mathematical model of the lateral motion was investigated by Pękalski [6] and improved by Kooi [4] and Zanevsky [8, 9].

We started analytical modelling of the system behaviour in the vertical plane using pseudo-static [2, 10] and dynamic approach [11]. There is a dynamic analysis of the traditional asymmetric bow in Japan [5] but it is very different of the modern sport bow and does not consider a human parameters. So, there are no analytical models, which make possibility to study archer, bow and arrow interaction in the vertical plane and improve calculated methods for adjustment of parameters of the system.

The aim of the research is to develop a method of mathematical modelling and computer simulation of static and dynamic interaction in the archer, bow and arrow system intending to get practical recommendations for the sport of archery.

Methods: of biomechanics (human body model as a system of mechanical oscillators); theoretical mechanics and mathematics (Lagrange equations of the second kind; the principle of d'Alembert; Cauchy problem, method of iterations and Runge-Kutta method); computing (MathCAD and Mathematica packets); experimental (high speed video analysis; nature mechanical modelling) [14-18].

Main results and discussion. In theoretical modelling, a human body is presenting as a mechanical system including a few solid bodies, which are connected one to other with viscous-elastic elements [12]. A model structure is build taking into consideration the aim of a research, body position and a character of interaction with surrounding. Sport archers stretch a bow during the time of common motion of a string and an arrow with fixed hand trying to keep steady pose of a body. The motor program of shooting is initiated before the clicker (a signal to release) can be heard [1].

Bow and archer interaction is located in the point of contact (*H*) of a bow hand and a bow handle (Fig. 1). Because a small time of string and arrow common motion (0,01-0,02 s) and relatively deep contact along the inner side of the thumb muscle, we can assume the point of contact (*H*) as a pivot point. Because small displacement of the bow handle, we can reduce the motion to orthogonal directions, which are suitable to describe bow hand motion in the vertical plane: they are longitudinal axis (μ - along the upper extremity) and perpendicular to it ($H\eta$) axis.

The next step to a suitable model is a kinematical linkage group with lower pairs (*E-D-H*). The number of freedom in the longitudinal direction is two: i.e. displacement of the body relatively a ground (rotation motion of the point *D* relatively the point *E*) and displacement of the upper extremity (contacting a bow handle) relatively the body (*HD*). The number of freedom in the perpendicular direction is one because elasticity in the shoulder girdle (*D*) is significantly bigger than its elasticity relatively the body in the vertical direction (*DE*).

Thus we can model an archer's body in the longitudinal direction using two particles and two viscous-elastic elements. The first particle with mass $m_{1\xi}$ models the archer's body (excepting the upper extremity contacting a bow handle) and has a virtual coordinate ξ_s. The first viscous-elastic element describes interaction in the body-ground system and has coefficients of stiffness $c_{1\xi}$ and viscosity $k_{1\xi}$. The second one, i.e. an upper extremity (of mass $m_{2\xi}$) has a virtual co-ordinate ξ_H because its common motion with a handle's contact point *H*. Corresponding viscous-elastic element with coefficients $c_{2\xi}$ and $k_{2\xi}$ describes interaction between the upper extremity and the rest part of the archer's body.

The model of motion in the perpendicular direction includes one particle (of mass m_η) with virtual coordinate η_H and viscous-elastic element with coefficients c_η and k_η.

Regarding the features stated above, we should write expressions for kinetic energy of the archer's body model (see Fig. 1):

$$T_{archer} = \frac{1}{2}\left(m_{1\xi}\xi_s'^2 + m_{2\xi}\xi_H'^2 + m_\eta \eta_H'^2\right);$$

potential energy:

$$P_{archer} = \frac{1}{2}\left[c_{1\xi}\xi_s^2 + c_{2\xi}\left(\xi_s - \xi_H\right)^2 + c_\eta \eta_H^2\right];$$

and dissipative function:

$$\Phi_{archer} = \frac{1}{2}\left[k_{1\xi}\xi_s'^2 + k_{2\xi}\left(\xi_s' - \xi_H'\right)^2 + k_\eta \eta_H'^2\right] \qquad (1).$$

Prefix shows a derivation in time, i.e. $(') \equiv d/dt$.

The basic assumption in modelling of a handle, limbs and an arrow is their motion as rigid bodies in a general (vertical) plane. A handle, a stabiliser and a sign could be modelled as one rigid body. The expression of its kinetic energy is (Fig. 2):

$$T_{handle} = \frac{1}{2}\int_{m_h}\left(\xi_h'^2 + \eta_h'^2\right)dm_h, \qquad (2)$$

where $\xi_h = \xi_H + y + x\kappa$; $\eta_h = \eta_H + x - y\kappa$; κ is an angular displacement relatively point *H*; *Hxy* is a rectangular system of co-ordinate fixed to the handle. Substituting the last two expressions in (2) we get:

$$T_{handle} = \frac{1}{2}\left[m_H\left(\xi_H'^2 + \eta_H'^2\right) + I_H\kappa'^2 + 2m_H\kappa'\left(\xi_H' x_{CH} - \eta_H' y_{CH}\right)\right], \qquad (3)$$

where m_H is mass; x_{CH}, y_{CH} are co-ordinates of a centre of gravity; I_H is moment of inertia relatively the point *H*.

An expression for kinetic energy of limbs is:

$$T_{U/L} = \frac{1}{2}\int_0^{l_{U/L}}\mu\left(z_{U/L}\right)\left(\xi_{U/L}'^2 + \eta_{U/L}'^2\right)dz_{U/L},$$

where $l_{U/L}$ is length of a limb; μ is distributed mass; z is co-ordinate fixed to a limb. The subdivides "U" and "L" mark corresponding the upper and lower limbs. Placing expressions of displacements

$$\xi_{U/L} = \xi_H \pm \kappa h_{U/L} + z_{U/L}\sin(\theta_{U/L} \pm \kappa);\; \eta_{U/L} = \eta_H \pm z_{U/L}\cos(\theta_{U/L} \pm \kappa)$$

to the last expression of energy $T_{U/L}$ we get:

$$T_{\lim bs} = \frac{1}{2}\left\{\begin{array}{l} \left(m_U + m_L\right)\left(\xi_H'^2 + \eta_H'^2\right) + \left(m_U h_U^2 + m_L h_L^2\right)\kappa'^2 + \\ I_U\left(\theta_U' + \kappa'\right)^2 + I_L\left(\theta_L' - \kappa'\right)^2 + \\ 2\left(m_U r_U - m_L r_L\right)\xi_H'\kappa' + 2\kappa'\begin{bmatrix} m_U r_U h_U\left(\theta_U' + \kappa'\right)\cos\left(\theta_U + \kappa\right) \\ - m_L r_L h_L\left(\theta_L' - \kappa'\right)\cos\left(\theta_L - \kappa\right)\end{bmatrix} + \\ 2\xi_H'\begin{bmatrix} m_U r_U\left(\theta_U' + \kappa'\right)\cos\left(\theta_U + \kappa\right) \\ + m_L r_L\left(\theta_L' - \kappa'\right)\cos\left(\theta_L - \kappa\right)\end{bmatrix} - 2\eta_H'\begin{bmatrix} m_U r_U\left(\theta_U' + \kappa'\right)\sin\left(\theta_U + \kappa\right) \\ - m_L r_L\left(\theta_L' - \kappa'\right)\sin\left(\theta_L - \kappa\right)\end{bmatrix} \end{array}\right\}, \qquad (4)$$

where $m_{U/L}$ - is mass of limbs; $I_{U/L}$ is moment of inertia of limbs relatively ends of the handle (points $H_{U/L}$); $r_{U/L}$ is distance from the end of the handle to the limb's centre of mass.

The expression of potential energy of limbs is (see Fig. 2 a):

$$P_{U/L} = \frac{1}{2}c_{U/L}\left(\theta_{U/L} + \varphi_{U/L}\right)^2, \qquad (5)$$

where; c_U, c_L are stiffness of limbs.

Expression of kinetic energy of a string is divided into three parts respectively three parts of string mass pinned to the nock points of the limbs and the arrow:

$$T_{string} = \frac{1}{2}\frac{m_s}{3}\left[\xi_A'^2 + \eta_A'^2 + \frac{2s_U^*}{s^*}\left(\xi_{TU}'^2 + \eta_{TU}'^2\right) + \frac{2s_L^*}{s^*}\left(\xi_{TL}'^2 + \eta_{TL}'^2\right)\right], \tag{6}$$

where m_s is mass of a string; $s_{U/L}^*$ is length of the string branches in un-stretched situations. $s^* = s_U^* + s_L^*$.

An expression of potential energy of string branches is:

$$P_{s(U/L)} = \frac{f\left(s_{U/L} - s_{U/L}^*\right)^2}{2s_{U/L}^*}, \tag{7}$$

where f is distributed stiffness of the string; $s_{U/L}$ i length of the string branches in stretched situations.

The expression of kinetic energy of the arrow is (see Fig. 2 b):

$$T_{arrow} = \frac{1}{2}\int_0^{l_a} \mu_a(z_a)\left(\xi_a'^2 + \eta_a'^2\right)dz_a + \frac{1}{2}m_P\left[\xi_a'^2 + \eta_a'^2\right]_{z_a = l_a}, \tag{8}$$

where l_a is length of the arrow; μ_a is distributed mass; z_a – co-ordinate fixed to its longitudinal axis; m_P is mass of the arrow's heard.

Bow and arrow interaction is described according the actual model only through contact at the nock point. Initial position of the arrow relatively the bow in its main (vertical) plane is determined with a rest that holds an arrow's head. A rest is fixed to the handle and has ability to turn and disappear just an arrow starts to move. Thanks a small size and mass, the rest does not accumulate energy therefore its interaction with an arrow is not taken into account in the frame of the model.

The results of high-speed video analysis show that we can assume arrow motion in the vertical plane as a rigid shift [7, 12]. Placing in the energy equation (8) the next expressions $\xi_a = \xi_A$; $\eta_a = \eta_A + z_a\psi$, we get:

$$T_{arrow} = \frac{1}{2}m_a\left(\xi_A'^2 + \eta_A'^2 + 2r_A\eta_A'\psi'\right) + I_A\psi'^2, \tag{9}$$

where m_a is mass of the arrow; r_A is a distance from the tail to the arrow's centre of mass.

The expression of potential energy of the arrow is calculated as a work of inertial forces (according the principle of d'Alembert) on the virtual longitudinal displacements ($\psi << 1$):

$$P_{arrow} = m_a r_A \left[\frac{1}{2} \xi''_A \left(\psi^2 - \psi_0^2 \right) + g(\psi - \psi_0) \right], \tag{10}$$

where ψ_0 is initial attitude angle of the arrow.

After some mathematical transformations in (1, 3-7, 9, 10), we get expressions for the whole kinetic and potential energy of the archer, bow and arrow system during their common motion between string release and arrow launch:

$$T = \frac{1}{2} \left\{ \begin{array}{l} \left(m_{2\xi} + m_H + m_U + m_L \right) \xi'^2_H + \left(m_\eta + m_H + m_U + m_L \right) \eta'^2_H + \\ \left(I_H + m_U h_U^2 + m_L h_L^2 \right) \kappa'^2 + 2 m_H \kappa' \left(\xi'_H x_{CH} - \eta'_H y_{CH} \right) + \\ I_U \left(\theta'_U + \kappa' \right)^2 + I_L \left(\theta'_L - \kappa' \right)^2 + 2 \left(m_U r_U - m_L r_L \right) \xi'_H \kappa' \\ + m_A \left(\xi'^2_A + \eta'^2_A \right) + 2 m_a r_A \eta'_A \psi' + I_A \psi'^2 + m_{1\xi} \xi'^2_s + \\ 2\kappa' \left[m_U r_U h_U \left(\theta'_U + \kappa' \right) b_1 - m_L r_L h_L \left(\theta'_L - \kappa' \right) b_3 \right] \\ + 2\xi'_H \left[m_U r_U \left(\theta'_U + \kappa' \right) b_1 + m_L r_L \left(\theta'_L - \kappa' \right) b_3 \right] \\ - 2\eta'_H \left[m_U r_U \left(\theta'_U + \kappa' \right) b_2 - m_L r_L \left(\theta'_L - \kappa' \right) b_4 \right] \end{array} \right\};$$

$$P = \frac{1}{2} \left[\begin{array}{l} c_{1\xi} \xi_s^2 + c_{2\xi} \left(\xi_s - \xi_H \right)^2 + c_\eta \eta_H^2 + c_U \left(\theta_U + \varphi_U \right)^2 + c_L \left(\theta_L + \varphi_L \right)^2 + \\ \frac{f}{\overset{*}{s}_U} \left(s_U - \overset{*}{s}_U \right)^2 + \frac{f}{\overset{*}{s}_L} \left(s_L - \overset{*}{s}_L \right)^2 + m_a r_A \left[\xi''_A \left(\psi^2 - \psi_0^2 \right) + 2g(\psi - \psi_0) \right] \end{array} \right], \tag{11}$$

where $b_1 = \cos(\theta_U + \kappa)$; $b_2 = \sin(\theta_U + \kappa)$; $b_3 = \cos(\theta_L - \kappa)$; $b_4 = \sin(\theta_L - \kappa)$; $m_A = \frac{1}{3} m_s + m_a$. Other two parts of string mass have been taken into account with mass-inertial characteristics of limbs as pinned to the nock points.

Solving the dynamic problem, we do not consider gravity forces acted the bow. But we take into consideration arrow weight because its force moment acted the arrow is the same value as the moment of inertial forces.

Placing expressions (1, 11) in the Lagrange equations of the second kind $\frac{d}{dt}\left(\frac{\partial T}{\partial q_i'}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial P}{\partial q_i}+\frac{\partial \Phi}{\partial q_i'}=F_i$, we get a system of differential equations of the second power relatively generated coordinates $q_i \equiv \xi_s, \xi_H, \eta_H, \kappa, \theta_U, \theta_L, \xi_A, \eta_A, \psi$:

$$m_{1\xi}\xi_s'' + c_{1\xi}\xi_s + c_{2\xi}(\xi_s - \xi_H) + k_{1\xi}\xi_s' + k_{2\xi}(\xi_s' - \xi_H') = F_{1\xi};$$

$$\begin{gathered}(m_H + m_U + m_L + m_{2\xi})\xi_H'' + c_{2\xi}(\xi_H - \xi_s) + k_{2\xi}(\xi_H' - \xi_s') + \\ m_U r_U\left[b_1(\theta_U'' + \kappa'') - b_2(\theta_U' + \kappa')^2\right] + m_L r_L\left[b_3(\theta_L'' - \kappa'') - b_4(\theta_L' - \kappa')^2\right]; \\ + (m_H x_{CH} + m_U h_U - m_L h_L)\kappa'' + e_U S_{U\xi} + e_L S_{L\xi} = F_{2\xi}\end{gathered}$$

$$\begin{gathered}m_U r_U\left[b_2(\theta_U'' + \kappa'') + b_1(\theta_U' + \kappa')^2\right] + m_L r_L\left[b_4(\theta_L'' - \kappa'') + b_3(\theta_L' - \kappa')^2\right] + \\ (m_H + m_U + m_L + m_\eta)\eta_H'' + c_\eta \eta_H + k_\eta \eta_H' - m_H y_{CH}\kappa'' - e_U S_{U\eta} + e_L S_{L\eta} = F_\eta\end{gathered};$$

$$\begin{gathered}(I_H + I_U + I_L + m_U h_U^2 + m_L h_L^2)\kappa'' + I_U \theta_U'' - I_L \theta_L'' \\ + m_U r_U h_U\left[b_1(\theta_U'' + 2\kappa'') - b_2(\theta_U' + \kappa')^2\right] - m_L r_L h_L\left[b_3(\theta_L'' - 2\kappa'') - b_4(\theta_L' - \kappa')^2\right] + \\ \left[m_H x_{CH} + m_U(h_U + b_1 r_U) - m_L(h_L + b_3 r_L)\right]\xi_H'' \\ - (m_H y_{CH} + m_U r_U b_2 + m_L r_L b_4)\eta_H'' + \\ + e_U\left[S_{U\xi}(b_1 l_U + h_U) - S_{U\eta} b_2 l_U\right] - e_L\left[S_{L\xi}(b_3 l_L + h_L) + S_{L\eta} b_4 l_L\right] = 0\end{gathered};$$

$$\begin{gathered}I_U(\theta_U'' + \kappa'') + m_U r_U h_U b_1 \kappa'' + m_U r_U(b_1 \xi_H'' - b_2 \eta_H'') \\ + c_U(\theta_U + \varphi_U) + e_U l_U(S_{U\xi} b_1 - S_{U\eta} b_2) = 0\end{gathered};$$

$$\begin{gathered}I_L(\theta_L'' - \kappa'') - m_L r_L h_L b_3 \kappa'' + m_L r_L(b_3 \xi_H'' + b_4 \eta_H'') \\ + c_L(\theta_L + \varphi_L) + e_L l_L(S_{L\xi} b_3 + S_{L\eta} b_4) = 0\end{gathered};$$

$$m_A \xi_A'' - e_U S_{U\xi} - e_L S_{L\xi} = 0;$$

$$m_A \eta_A'' + m_a r_A \psi'' + m_a g - e_U S_{U\eta} - e_L S_{L\eta} = 0;$$

$$I_A \psi'' + m_a r_A(\eta_A'' + \xi_A'' \psi + g) = 0, \qquad (12)$$

where F_i - are generated forces;

$$S_{U\eta} = \eta_H + h_U + l_U b_1 - \eta_A; \; S_{L\eta} = \eta_H - h_L + l_L b_3 - \eta_A;$$

$$S_{U\xi} = \xi_H + h_U \kappa + l_U b_2 - \xi_A; \; S_{L\xi} = \xi_H - h_L \kappa + l_L b_4 - \xi_A$$

$$e_U = \frac{f(s_U - s_U^*)}{s_U s_U^*}; \; e_L = \frac{f(s_L - s_L^*)}{s_L s_L^*};$$

$$s_U = \sqrt{S_{U\xi}^2 + S_{U\eta}^2}; \;\; s_L = \sqrt{S_{L\xi}^2 + S_{L\eta}^2}.$$

The initial conditions of the problem are:

$$\begin{aligned} &t = 0, \xi_s = 0; \xi_H = \xi_{H0}; \eta_H = \eta_{H0}; \xi_A = \xi_{A0}; \eta_A = \eta_{A0}; \theta_U = \theta_{U0}; \theta_L = \theta_{L0}; \kappa = 0; \\ &\psi = \psi_0; \xi_s' = 0; \xi_H' = 0; \eta_H' = 0; \xi_A' = 0; \eta_A' = 0; \theta_U' = 0; \theta_L' = 0; \kappa' = 0; \psi' = 0, \end{aligned} \tag{13}$$

where constants η_{A0}, θ_{U0}, θ_{L0}, ξ_{H0}, η_{H0} are the solutions of the static problem (see the next section). Zero values of derivations correspond the manner of sport archer technique (a breathing is stopped and a pose is motionless).

According the results of surface electromyography, the motor program of shooting is initiated before the signal to release can be heard. Thus it can be classified as open loop. The anticipation of the balance's release is characterized by an increasing activity of the m. pectoralis major only [1]. This muscle is activated to balance changes of static equilibrium in the lateral plane. No muscles show significant changes of activity. Therefore, no generated forces should be present in the Lagrange equations (12), which describe behaviour of the system in the vertical plane: $F_i = 0$.

The direction of arrow motion (the vector of the centre of mass) is described with expressions below:

$$tg\zeta = \frac{\eta_A' + r_A\psi'}{-\xi_A'}; \; \alpha = \psi - \zeta, \tag{14}$$

where ζ is the angle of projection, and α is the angle of attack. An arrow leaves a string just after its acceleration equal zero $(\xi_A'' = 0)$.

The system of equations (12) with initial conditions (13) presents a Cauchy problem for non-linear ordinary differential equations of the second power. It is impossible to get analytical solutions for the problem therefore we used Runge-Kutta method applied in the program 'NDSolve' (Method Explicit Runge Kutta) from the package Mathematica 4.1 (www.wolfram.com).

This mathematical model describes dynamics of the bow and arrow system in the vertical plane (or, if to use a rigorous style, bow and arrow common motion in the main plane of the bow). Really, the system makes 3D motion, but its displacement in the lateral plane is hundred times smaller than displacement in the main plane. The influence of the lateral motion to the main motion is negligible small because significantly different value of displacement and correspondent energy [7].

The initial conditions regarding the pivot point displacements in bow hand and handle interaction are:

$$\xi_{H0} = \frac{F_\xi}{c_{2\xi}}; \;\; \eta_{H0} = \frac{F_\eta}{c_\eta} \tag{15}$$

where $F_\xi = -F_U \sin\gamma_U - F_L \sin\gamma_L; F_\eta = F_U \cos\gamma_U - F_L \cos\gamma_L$; F_ξ, F_η are projections of the drawn force to the axes of co-ordinates; F_U, F_L are forces in string branches.

The static problem includes transcendent and non-linear functions, and they cannot be solved analytically [13]. Therefore to study of the static problem, we applied numerical method of iteration using the computer program 'Find' from the package Mathcad 2000i Professional (www.mathcad.com).

Let's consider a modern sport bow and arrow made in the frames of FITA (International Archery Federation) Standard (www.archery.org) that has media parameters: WIN&WIN Recurve Bow. The bow consists of Winact Riser (25") and Long Limbs (70"), i.e. bow handle length is h*=635 mm and the whole length of the bow (measured between tips of the limbs) is 1778 mm. The standard measure of bow asymmetry in the vertical plane named 'tiller' is $\Delta = 6$ mm and bow force is $F = 178$ N. The arrow No 2414 - 30" with 15% mass point was used.

Rated parameters of the bow are (see previous sections): $l_U = l_L = 531$ mm; $m_U = 106$ g; $m_L = 107$ g; $I_U = 68{,}1$ kgcm^2; $I_L = 68{,}3$ kgcm^2; $r_U = 227$ mm; $r_L = 228$ mm; $c_U = c_L = 69{,}1$ Nm; $\varphi_U = 0{,}6047$; $\varphi_L = 0{,}6076$ $h_U = h_L = 342$ cm; $m_H = 2{,}13$ kg; $I_H = 2128$ kgcm^2; $x_{CH} =$ -21 mm; $y_{CH} =$ -34 mm; $s_U^* = 780$ mm; $s_L^* = 840$ mm; $f = 255$ N/cm; $m_s = 7$ g; $\xi_{A0} = 0{,}7576$.

Bow parameters in the braced position are: $\theta_{UB} = 0{,}4666$; $\theta_{LB} = 0{,}4342$; $s_B = 820$ mm; $\gamma_B = 0{,}00945$; $F_B = 316$ N, where s_B is the string length; F_B is the force of the string stretch. Subdivides «B» mark parameters of a bow with a braced string. The arrow parameters are: $l_a = 783$ mm; $m_a = 22{,}4$ g; $I_A = 73{,}6$ kgcm^2; $r_A = 510$ mm, where $l_a = \xi_A$ is a drawn distance determined by the length of an arrow. The male archer subject parameters are: $m_{1\xi} = 26$ kg; $m_{2\xi} = 3{,}8$ kg; $m_\eta = 2{,}1$ kg; $c_{1\xi} = 11{,}6$ N/mm; $c_{2\xi} = 19{,}0$ N/mm; $c_\eta = 9{,}43$ N/mm; $k_{1\xi} = 237$ kg/s; $k_{2\xi} = 78$ kg/s; $k_\eta = 45$ kg/s.

Solving the static problem we get data parameters in the initial conditions (13) and from (15) too: $\eta_{A0} = 42{,}6$ mm; $\theta_{U0} = 0{,}7655$; $\theta_{L0} = 0{,}7942$; $\xi_{H0} = 9{,}4$ mm; $\eta_{H0} = 1{,}1$ mm. The other solutions are: $s_U = 786$ mm; $s_L = 846$ mm; $\gamma_U = 0{,}5189$; $\gamma_L = 0{,}4641$; $F_U = 186$ N; $F_L = 192$ N; $F_\xi = 178$ N; $F_\eta = 10$ N.

The results of archer, bow and arrow dynamics during their common motion, as the initial rest height $\eta_{P0} = 37$ mm, are presented on the graphs (Fig. 3, 4). A time of bow and arrow common motion (from string release up to arrow launch) that is about 0,0154 s was calculated taking into account the instant of the maximum longitudinal speed ($V_A = 62$ m/s) as zero value of relating acceleration ($\xi''_A = 0$, see Fig. 4 a). A recoil force acted between a bow hand and a handle after string release varies in the range near 3 % about its initial (static) value (see Fig. 3 c). A force, reduced near 20 times (see Fig. 3 b), is translated to the ground via an archer's body wile its displacement remains near 1 mm (see Fig. 3 a).

Because different character of static and dynamic balances of forces, an arrow launches a string a few millimetres deeper ($\xi_A = 223$ mm, see Fig. 4 b) than it is in the braced position ($\xi_A = 231$ mm). Displacement of the pivot point, i.e. a point of contact of a bow hand and a handle, is about one millimetre and approximately is equal in the longitudinal ($9{,}4 < \xi_H < 10{,}2$ mm) and perpendicular directions (see Fig. 4 d).

String and arrow common motion (internal ballistics) is accompanied with intensive oscillations, which are caused by destruction of the static balance of forces at the instant of string release. There are seven full cycles of oscillation during the motion. The results of computer simulation make possible to determine bow and arrow adjusted parameters, which minimize the angle of attack and angular speed of an arrow that is better for a good shot.

Conclusions

The results of modelling of the archer-bow-arrow system correlate with well-known results of high-speed video analysis: the process of common motion has significant non-linear character.

A recoil force, acted between a bow hand and a handle, after string release varies in the range near 3 % about its initial (static) value. A force reduced near 20 times is translated to the ground via an archer's body wile its displacement remains near 1 mm.

Because different character of static and dynamic balances of forces, an arrow launches a string a few millimetres deeper than it is in the braced position.

String and arrow common motion (internal ballistics) is accompanied with intensive oscillations, which are caused by destruction of the static balance of forces at the instant of string release. There are seven full cycles of oscillation during the motion.

Acknowledgements. The author is grateful to the staff of Physical and Health Education Department, Casimir Pulaski Technical University, Radom for close and fruitful collaboration.

References

Edelmann-Nusser J., Gollhofer A. Coordinative aspects of archery – an approach using surface electromyography. XVI Int. Symp. Biomechanics in Sports. Ed.: H.J. Riehle, M.M. Vieten. Proc. II. Konstanz: UVK, 1998. 153-156.

Gros H., Zanevskyy I. Archer-bow-arrow system adjustment in the vertical plane. Scientific proceeding of the XX International Symposium of Biomechanics in Sports, Ed. K.E.Gianikellis, Universided de Extremadura, Spain, 2002, 469-472.

Hickman C.N., Nagler F., Klopsteg P.E. Archery: the technical side. – Redlands: National Field Archery Association, 1947. – 148 p.

Kooi B.W. Bow-arrow interaction in archery. Journal of Sports Sciences. – 1998. – 16. – P. 721-731.

Ohsima S. & Ohtsuki A. Simulation of the shape and dynamics of Japanese bow – Application of large deflection theory. The book of the 4^{th} Int. conf. on the Engineering of Sport, Kyoto, Japan, 2002, 102-107.

Pękalski R. Modelling and simulation research of the competitor-bow-arrow system. Unpublished doctoral dissertation. – Warszawa: AWF, 1987.

Werner Beiter Zeigt. Highspeed film. – 1992.

Zanevskyy I. Dynamics of "arrow-bow" system. Journal of Automation and Information Sciences. – 1999. - 31 (3). - pp. 11-17

Zanevskyy I. Lateral deflection of archery arrows. Sports Engineering.- 2001.- Vo 4. - No 1. - P.23-42.

Заневський І.П. Технічний аспект налаштування спортивного лука / Технічні вісті, 2001, 1 (12), с. 24-29.

Zanevskyy I., Ohirko I. Mechanical and mathematical modelling of bow and arrow interaction. Acta of Bioengineering and Biomechanics, 2002, 4, Suppl. 1, 615-616.

Заневський І.П. `Методика моделювання та аналізу параметрів пострілу зі спортивного лука. Дис. наук. ступ. д. т. н. спец. 05.13.02. ФМІ НАН України, 1996.

Заневский И.П. Математическое и компьютерное моделирование системы "спортсмен-лук-стрела". // Наука в олимпийском спорте. 2005.№2. – С.128-136.

Заневський І.П. Динаміка системи лук-стріла у вертикальній площині. // Машинознавство, 2003, № 8, (74) - С. 7-15.

Zanevskyy I. A model of string-limbs stiffness in the lateral plane of the sport bow. The engineering of sport 4, Ed. S.Ujihachi & S.J.Haake, Blackwell Publishing, Oxford, UK, 2002, 65-71.

Zanevskyy I. Mechanical and mathematical modelling and computer simulation of vibration and impact processes in the "Man and Shooting Device" system / Virtual Non-linear Multibody Systems. – NATO Advance Study Institute, Vo 1, Prague, 2002, 248-252.

Заневський І.П. Налаштування параметрів спортивного лука у вертикальній площині / Збірка наукових праць 2-ї міжн. конф. ‘FITA-70’, 2001. - с. 10-22.

Zanevskyy I. String materials regarding bow and arrow dynamics / A collection of papers “Materials and Science in Sports”, Coronado, California, USA, 2001, 83-93.

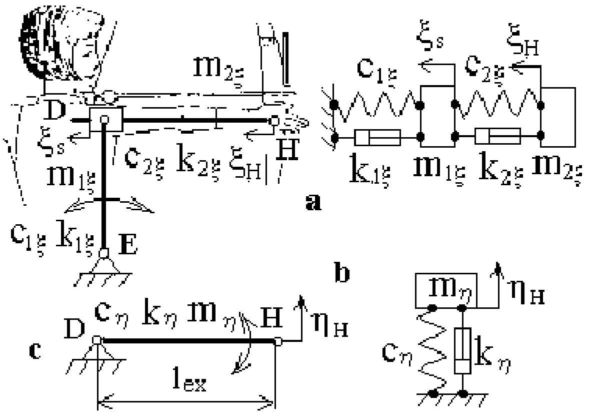

Fig. 1. A model of the archer’s body: a – longitudinal direction; b – perpendicular direction.

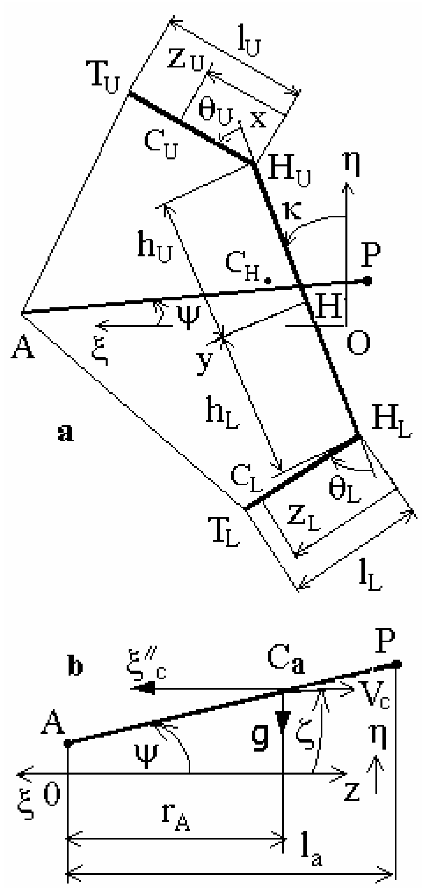

Fig. 2. Dynamic scheme models of a bow (a) and an arrow (b).

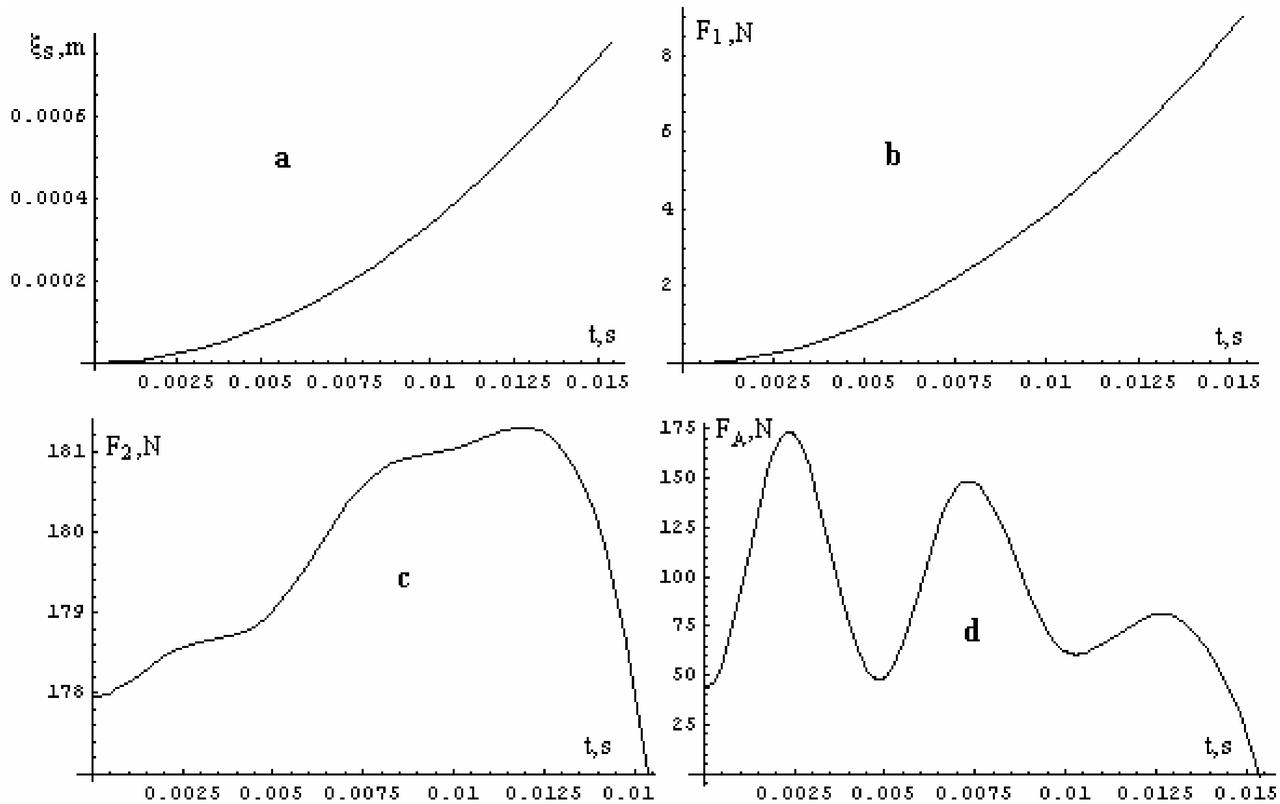

Fig. 3. Parameter of bow and archer interaction: a – virtual displacement of the archer body; b – recoil force acted to the body; c – recoil force acted to the bow hand.

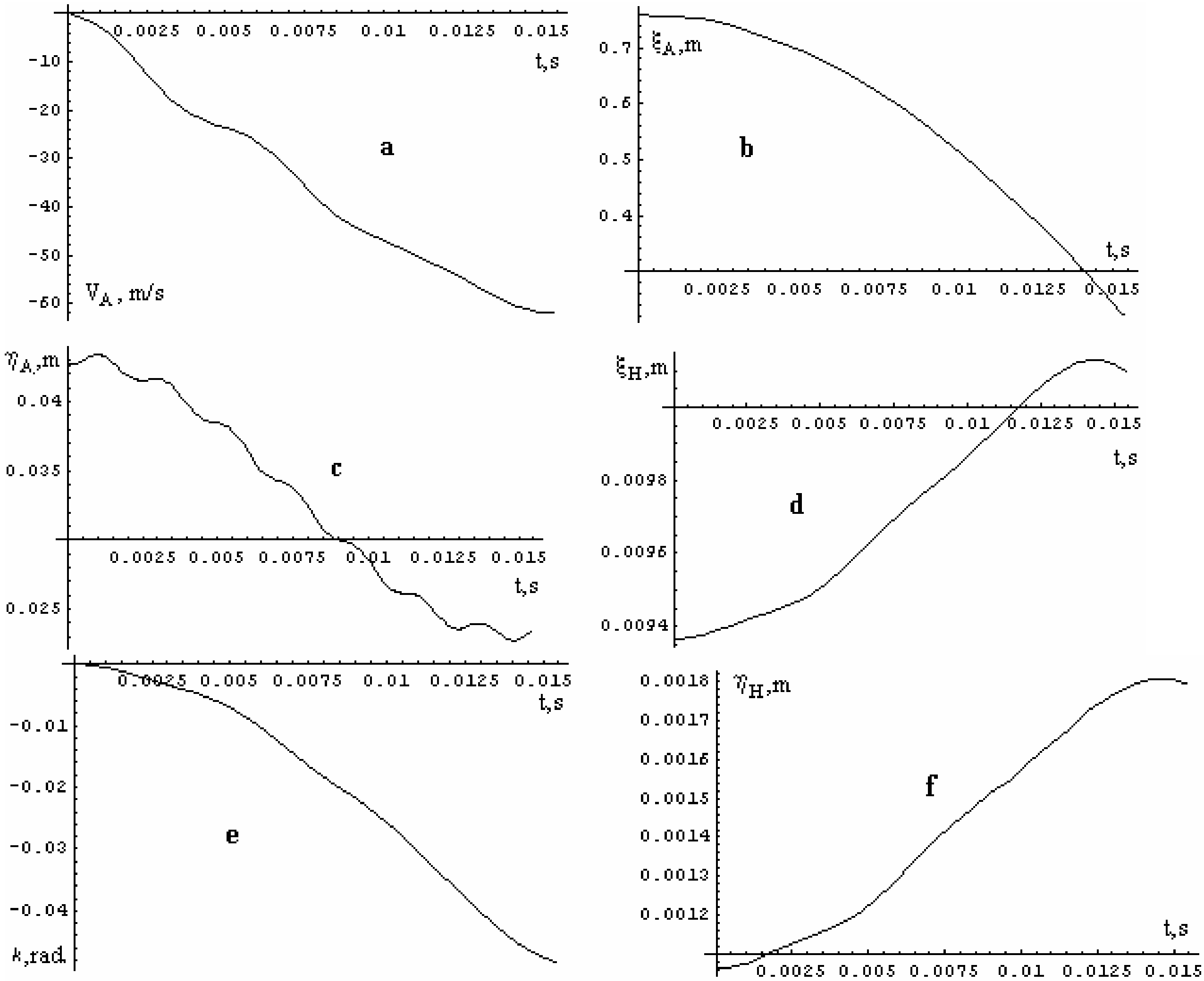

Fig. 4. Parameter of bow and arrow interaction: a – longitudinal speed of an arrow; b – longitudinal displacement of an arrow; c – perpendicular displacement of string and arrow nock point; d - longitudinal displacement of the pivot point (hand and handle); e – angular displacement of a handle.